1. INTRODUCTION

Cloud droplet size distributions — hence the key microphysical quantities (e.g., radar reflectivity, droplet concentration, liquid water content, relative dispersion, and mean-volume radius) are determined by different physical mechanisms, including pre-cloud aerosols as CCNs, cloud updraft, and various turbulent entrainment-mixing processes. Therefore, different relationships among these microphysical properties are expected in response to these various mechanisms. The effect of turbulent entrainment-mixing processes is particularly vexing, with different entrainment-mixing processes likely leading to different microphysical relationships.

Cloud radar has been widely used to infer the cloud liquid water content \((L)\) from the measurement of radar reflectivity \((Z)\) using a Z-L relationship. Existing Z-L expressions have been often obtained empirically, and differ substantially (Khain et al. 2008). The discrepancy among Z-L relations, which has been hindering the application of cloud radar in measuring cloud properties, likely stems from the different relationships between the relevant microphysical properties caused by different physical processes.

This study first analyzes the Z-L relationship theoretically, and identify the key microphysical properties that affect this relationship, and then address the effects of various processes on the Z-L relationship by discerning the characteristics of the relationships between the relative dispersion, droplet concentration, liquid water content, and mean-volume radius calculated from in-situ measurements of cloud droplet size distributions. Effort is also made to further relate the microphysical relationships to physical processes such as turbulent entrainment-mixing.

2. THEORETICAL ANALYSIS

Define the \(p\)-th mean radius \(r_p\) as

\[
r_p^p = \left( \frac{\int r^n n(r) dr}{N} \right)^{1/p},
\]

where \(r\) is the droplet radius, \(n(r)\) the droplet concentration per unit \(r\) interval, and \(N\) the total droplet concentration. Radar reflectivity \(Z\) can be expressed as

\[
Z = 64 \int r^n n(r) dr = 64 N r_6^n
\]

(2a)

Equation (2a) can be rewritten in two different forms:

\[
Z = \frac{36}{\pi^2} \frac{\beta_6^6}{L^2} \frac{L^2}{N},
\]

(2b)

\[
Z = \frac{3 \times 64}{4 \pi \rho_w} \beta_6^6 r_6^3 L,
\]

(2c)

The following two results are expected from Eqs. (2b) and (2c) respectively:

1) For adiabatic clouds with constant \(N\) and \(\beta_6\) as commonly assumed (Atlas, 1954)
\[ Z \propto \beta_6^\alpha L^2 = BL^2 \propto L^2. \]  

Equations (3a) and (3b) cover only two ideal scenarios: there exist many different mechanisms in reality that likely lead to different microphysical relationships among \( \beta_6, r_3, N, \) and \( L, \) and hence different Z-L relationships. Accordingly, if Z and L follow a power-law relationship given by

\[ Z = aL^\gamma, \]  

scenarios with \( \gamma \) differing from 1 or 2 can not be ruled out, and moreover, the specific value of \( \gamma \) is expected to depend on the acting physical mechanisms. This simple analysis reveals the potentially important roles of \( \beta_6, r_3, N \) and their relationships to \( L \) in determining the specific Z-L relation and its relevance to the physical processes at work. It is noteworthy that \( \beta_6 \) actually measures the relative width of the cloud droplet size distribution because \( \beta_6 \) generally is an increasing function of the relative dispersion (e.g., Liu and Daum, 2000, 2004). As will become evident, \( \beta_6 \) is very useful in identifying the different entrainment-mixing mechanisms, although its role has been ignored in the common assumption of a constant \( \beta_6. \)

3. **Observational Results**

To examine the effects of aerosol particles on cloud microphysics, the DOE Atmospheric Sciences Program (ASP), the DOE Atmospheric Radiation Measurements Program (ARM), and the Naval Research Laboratory’s Center for Remotely Piloted Vehicles for Atmospheric Studies (CIRPAS) conducted a joint field campaign in the 2005 summer, The Marine Stratus/Stratocumulus Experiment (MASE), to investigate the marine stratus/stratocumulus clouds that commonly occur off the coast of Northern California (ARM deployed its mobile facility at Pt Reyes National Seashore just North of San Francisco as part of the Marine Stratus Radiation, Aerosol and Drizzle experiment (MASRAD). The DOE G-1 aircraft carried a suite of instrumentation to measure aerosol composition and microphysical properties of aerosols and clouds, state parameters, winds, and radiation fields. Here we report results derived from the cloud droplet size distributions from 0.5 to 25 \( \mu \)m in radius measured with a Cloud Aerosol and Precipitation Spectrometer (CAPS) (Droplet Measurement Technologies, Boulder, CO). This two-section instrument measures droplets in the 0.5 – 25 \( \mu \)m diameter range using a light scattering technique, and droplets in the 25 – 1550 \( \mu \)m diameter range using an imaging technique (see Daum et al. 2008; Lu et al, 2007 for details about MASE). A segment of data from a horizontal flight during ACE1 is also analyzed in view of its long sampling period, wide range of variability, and features of homogeneous mixing processes.

3.1. **Horizontal-Vertical Contrast**

Cloud microphysics is affected by different mechanisms in horizontal and vertical directions, and these differences are expected to manifest themselves in the Z-L relationship. This general expectation should hold for horizontal legs at different altitudes and in different clouds as well. To confirm this, we partition the data into horizontal legs and vertical profiles (averages of all the horizontal legs at different altitudes), and then compare them. As expected, substantial differences are found. The contrasts between horizontal and vertical results are especially striking, generally with \( \gamma < 1 \) for the vertical profiles but \( \gamma > 1 \) for the corresponding horizontal legs. Figure 1 shows an example derived from flight 20 July 2005a. It is clear that generally the slopes of the horizontal legs...
are steeper than that of the vertical profile. This result reinforces the need to distinguish between horizontal and vertical variations in studies of cloud properties. See next for more discussions on the physics underlying this striking difference.

3.2. Occurrence Frequency of \( \gamma \)

A total of 35 horizontal cases and 12 vertical profiles are analyzed for the Z-L power-law fit given by Eq. (4) for the MASE data. Figure 2 depicts the occurrence frequency of different values of \( \gamma \) for the 47 cases. It is evident that \( \gamma \) ranges from less than 1 to larger than 2, although it peaks between 1 and 2. It is noteworthy that many cases with \( \gamma < 1 \) are for the vertical profiles, a surprising result in view of the conventional wisdom that in vertical direction, adiabatic condensational growth dominates and \( \gamma \) should be close to 2 according to Eq. (3a).

3.3. Case Studies

Here three special cases that span the typical range of \( \gamma \) values are examined in details. First, according to Eq. (3b), \( \gamma < 1 \) requires a negatively correlated relation between the parameter A and L. This can happen when some inhomogeneous mixing is in action whereby both \( \beta_6 \) and \( r_3 \) decrease with increasing L, i.e., spectral broadens toward large droplets, in expense of evaporation of small droplets (Daum et al., 2008). This scenario can also happen with a proper combination of changes of \( \beta_6 \) and \( r_3 \) with L. Figure 3 shows an example of the latter with \( \gamma = 0.73 \). Evidently, the parameter A decreases with increasing L when L is small, and this decrease is caused by a relatively faster decreasing rate of \( \beta_6 \) compared to the increasing rate of \( r_3 \).

It is expected that the other cases of the vertical profiles with \( \gamma < 1 \) are due to the same reason because of condensational growth (positive \( r_\gamma \)L correlation) and narrowing spectra (negative \( \beta_6 \)L correlation) with increasing altitudes above cloud bases.

Figure 4a shows an example of Z-L relationship with \( \gamma = 1.31 \), along with the corresponding A and B parameter. Unlike the case with \( \gamma < 1 \) shown in Fig. 3, which is
due primarily to condensational growth and spectral narrowing with increasing heights, this case is likely associated with the homogeneous entrainment-mixing process. The value $\gamma$ is larger than 1 because of the increase of A but the decrease of B with increasing L.

As further shown in Fig. 4b, the behavior of A occurs because, although the changes of $\beta_6$ and $r_3$ with L are similar to the first case, the increase of $r_3^3$ with L is steeper than the decrease of $\beta_6^6$, resulting a positive dependence of A but a negative dependence of B on L and $1 < \gamma < 2$. Note that A is proportional to BL; a slope of A less than 1 guarantees a negative slope of B. This is why the B curve is omitted in Fig. 3.

Figure 4b. Variation of $\beta_6$, N and $r_3$ with L. The colors of the symbols correspond to those of the axes.

Figure 5b shows the relationships of $r_v$, $\beta_6$, and N to L. Strikingly, $\beta_6$ increases with increasing L but N decreases with increasing L for most large values of L. This behavior is unexpected from the traditional views of homogeneous or inhomogeneous or ETEM mixing mechanisms because the opposite is expected from all of them. There are two plausible mechanisms for this. First, when L is large, collection process kicks in, leading to larger dispersion and smaller N. However, the small value of $r_3$ seems contradictory to this hypothesis. The second possibility is that the mixed parcel with reduced N and enhanced dispersion further mixes with parcels with large L. Clearly, more needs to be done about this scenario.

Figure 5a depicts an example with $\gamma > 2$ derived from the flight leg at altitude of 260 m, 15 July 2005, during the MASE experiment. In contrast to the previous two scenarios, the striking feature of this case is that the parameter B increases with increasing L, inducing a $\gamma > 2$ according to Eq. (2b). Again the A curve is omitted because a positive B-L correlation guarantees a positive A-L correlation.

Figure 5a. An example of Z-L relation with $\gamma = 3.67$. The colors of the symbols correspond to those of the axes.
4. CONCLUDING REMARKS

Theoretical analysis reveals that the commonly assumed two scenarios about the Z-L relationship only represent two ideal cases, and that the real scenario depends on combined dependences of $r_v$, $\beta_6$, and $N$ on $L$. Empirical examination of the data derived from measured droplet size distributions further shows the possibility for the power exponent $\gamma$ in the Z-L relation to range from less than 1 to larger than 2. Three examples with $\gamma = 0.73$, 1.31, and 3.67 are analyzed in detail from the perspective of the relationships of $r_v$, $\beta_6$, and $N$ to $L$. The results indicate that the three examples likely arise from condensational growth together with narrowing with increasing heights, homogenous mixing, and inhomogeneous/ITEM mixing/collection processes, respectively.

Although the results are preliminary and much remains to be explored, this work does point to the close link between the Z-L relations and the relationships of $r_v$, $\beta_6$, and $N$ to $L$ that have been long overlooked. The important role of the spectral shape needs to be re-emphasized as it has been largely ignored in remote sensing studies of cloud properties. This work also suggests a potential remote sensing approach to address the outstanding problem of entrainment-mixing mechanisms, which are mainly limited to using aircraft measurements so far. Of course, more analyses are necessary to establish a solid basis for the link between the Z-L relationship and various turbulent entrainment-mixing processes. Such kind of research is underway.

REFERENCES


ACKNOWLEDGEMENT: This research is supported by the DOE ARM program and ASP program.