1. INTRODUCTION

The bright band, a layer of enhanced radar echo associated with melting hydrometeors, is often observed in stratiform rain. Understanding the microphysical properties of melting hydrometeors and their electric scattering and propagation effects is of great importance in accurately estimating parameters of the precipitation from spaceborne radar and radiometers, such as TRMM PR and TMI and future GPM DPR and GMI (Bringi et al. 1986; Fabry and Szymer 1999; Olsen et al., 2001a and 2001b; Meneghini and Liao 2000; Liao and Meneghini 2005). However, one of the most difficult problems in the study of the radar signature of the melting layer is the determination of the effective dielectric constants of melting hydrometeors. Although a number of mixing formulas are available to compute the effective dielectric constants, their results vary to a great extent when water is involved in the mixture, such as in the case of melting snow. It is physically unclear as to how to select among these various formulas (Meneghini and Liao 1996).

Although some success was achieved in simulating the radar bright-band signatures from the TRMM Precipitation Radar (Ku band) and airborne dual-wavelength radar (X and Ka bands) by modeling melting snow as a stratified sphere, a sphere composed of multiple layers (Liao and Meneghini 2005), the accuracy of the formulation needs to be examined in greater detail by means of radar measurements at other frequencies. Simultaneous measurements of the bright band made by the EDOP (X-band) and CRS (W-band) airborne Doppler radars during the CRYSTAL-FACE campaign in 2002 provide an excellent opportunity to check the validity of the stratified-sphere scattering model. Measurements of both radar reflectivities and Doppler velocities at two frequencies with the higher frequency at W-band are particularly useful for testing the model. In the stratified-sphere model the water fraction is constant in each layer of the stratified sphere but is allowed to vary from layer to layer. As such, the stratified-sphere scattering model can be used to compute scattering parameters for non-uniformly melting hydrometeors whose fractional water content is prescribed as a function of radius of sphere. In conjunction with a melting layer model that describes the melting fractions and fall velocities of hydrometeors as a function of the distance below the 0 °C isotherm, the radar bright-band profiles can be simulated for airborne radars.

The paper is organized as follows. In Section 2 we derive the effective dielectric constants of uniformly mixed snow and water particles from their internal electric fields by using the computational model in which the particles are described by a collection of 128x128x128 cubic cells of identical size and the CGFFT (Conjugate Gradient Fast Fourier Transform) numerical method. Procedures to simulate the radar bright-band signatures by use of the stratified-sphere model are described in Section 3. Comparisons of the simulated radar profiles in the melting layer of the EDOP and CRS airborne measurements are given in Section 4 followed by the summary in Section 5.

2. EFFECTIVE DIELECTRIC CONSTANT

Let \( E(r, \lambda) \) and \( D(r, \lambda) \) be the local electric and dielectric displacement fields at free-space wavelength \( \lambda \), satisfying

\[
D(r, \lambda) = \varepsilon(r, \lambda) E(r, \lambda),
\]

where \( \varepsilon \) is the dielectric constant. In view of the local constitutive law described by the above equation, the bulk effective dielectric constant, \( \varepsilon_{\text{eff}} \), at sufficiently long wavelength is defined as (Stroud and Pan 1978)
\[ \varepsilon_{\text{eff}} \iiint_V E(r, \lambda) dv = \iiint_V D(r, \lambda) dv \]  \hspace{1cm} (2)

If the particle, composed of two materials, \( \varepsilon_1 \) and \( \varepsilon_2 \), is approximated by \( N \) small equal-volume elements, then the \( \varepsilon_{\text{eff}} \) can be written as

\[ \varepsilon_{\text{eff}} = \frac{\varepsilon_1 \sum_{j=1}^{M_1} E_j + \varepsilon_2 \sum_{j=1}^{M_2} E_j}{\sum_{j=1}^{M_1} E_j + \sum_{j=1}^{M_2} E_j} \]  \hspace{1cm} (3)

The notations \( \sum_{j=1}^{M_1} \) and \( \sum_{j=1}^{M_2} \) denote summations over all volume elements comprising materials 1 and 2, respectively. In this study, the internal fields appearing on the right-hand sides of (3) are computed by the CGFFT numerical procedure in which the volume enclosing the total particle is divided into \( 128 \times 128 \times 128 \) identical cells. Validation of the computational procedures for \( \varepsilon_{\text{eff}} \) has been extensively carried out for uniform and non-uniform snow-water mixtures (Meneghini and Liao, 1996 and 2000; Liao and Meneghini, 2005). This is done by comparing the scattering parameters, such as backscattering and extinction cross sections, and phase function, from realizations of the mixed-phase particle models with those from a uniform particle with dielectric constant \( \varepsilon_{\text{eff}} \). It has been shown that \( \varepsilon_{\text{eff}} \) as derived from (3) is sufficiently accurate to compute the effective dielectric constant of snow and water mixtures in the microwave range.

Figures 1 and 2 display the real and imaginary parts of \( \varepsilon_{\text{eff}} \) of homogeneous snow-water mixtures versus water fractions at X and W bands as computed from (3) by the CGFFT. The computations are made for a snow density of 0.1 g/cm\(^3\). For comparisons, the results from the Maxwell Garnett (1904) and the Bruggeman (1935) mixing formulas are also given in the plots. An example of a realization of a uniformly mixed snow-water particle is shown in Fig.3 for a water fraction of 0.3. The dark and light gray areas...
represent water and snow, respectively. The minimum size of any snow or water region is chosen to be at least $4 \times 4 \times 4$ cells to better satisfy the boundary conditions at the snow-water interfaces. As can be seen in Figs. 1 and 2, the results of $\varepsilon_{\text{eff}}$ derived from the CGFFT lie between the two results derived from the Maxwell Garnett mixing formula, one in which water is treated as the matrix with snow inclusions ($\text{MG}_{\text{WS}}$), and the other in which the roles of water and snow are reversed, i.e., snow as matrix and water as inclusion ($\text{MG}_{\text{SW}}$). The results of the Bruggeman’s mixing formula are also bounded within the results of $\text{MG}_{\text{WS}}$ and $\text{MG}_{\text{SW}}$, but tend to yield larger real and imaginary parts of $\varepsilon_{\text{eff}}$ than the CGFFT.

3. BRIGHT-BAND SIMULATIONS

To simulate the radar signatures in the melting layer, two models are required: One is the melting layer model that provides microphysical properties of the mixed-phase hydrometeors, such as melting fractions and fall velocities of individual hydrometeors over their size spectra, as a function of the distance from $0^\circ \text{C}$ isotherm; the other is the particle scattering model that is used to compute the scattering properties of melting hydrometeors. Using the information provided by the melting layer model along with the particle scattering model, snow mass density and particle size distribution, the backscattering intensities and attenuation coefficients can be computed from any location within the melting region. In this study, the snow falls and melts in accordance with the model described by Yokoyama and Tanaka (1984). Aggregation and drop breakup are not included in the model. To model the fact that melting usually starts at the particle surface and then progresses toward the center, we employ the stratified-sphere particle model, which consists of 100 concentric equal-thickness layers. The melting water distribution or fractional water content inside the particle can be expressed as a function of radius. Within each layer of the stratified sphere the effective dielectric constant is fixed and determined from the results of Figs. 1 and 2 (X and W bands, respectively) based on the fractional water content specified at the layer of interest. An exponential function is adopted to describe the fractional water content $f_W$ in terms of radius $r$:

$$f_W(r) = \begin{cases} \frac{\beta r}{r_0} & \text{for } r < r' \\ 1 & \text{for } r_0 \geq r > r' \end{cases}$$

(4)

where $r_0$ is the radius of the particle, and $r'$ is the radius at which $f_W$ is equal to 1, i.e., $f_W(r')=1$. The coefficient $\beta$ specifies the radial gradient of the water fraction so that a larger $\beta$ results in a more rapid transition from snow to water. Its value was found to be 4.5 from the simulation study reported by Liao and Meneghini (2005). Shown in Fig. 5 are the simulated results of the X- and W-band radar.
profiles in the melting layer for snow densities of 0.05, 0.1 and 0.2 g/cm$^3$ as computed from the melting layer model and stratified-sphere scattering model described above. In these simulations the Marshall-Palmer raindrop size distribution (1948) is assumed for a rain rate of 1.5 mm/h. The attenuation due to hydrometeors is also taken into account in the results. A change in the snow density has different impact on the results of the simulated bright-band profiles at X and W bands. The smallest snow density ($\rho=0.05$ g/cm$^3$) gives the biggest enhancement of the reflectivity at X band but yields the narrowest bright-band width. At W band no clear radar bright bands are seen in Fig.5, even though a strong enhancement in the radar reflectivity is apparent in the early stages of melting. In contrast to the results at X band, the biggest change in the radar reflectivity at W band from snow to the bright-band peak occurs at $\rho=0.2$ g/cm$^3$, the highest snow density among those used in the plot. After reaching the maximum, the radar reflectivities computed from all the values of the snow density tend to converge, and their intensities remain nearly constant up to the rain region. It should be noted that the primary difference in the bright-band signatures at these frequencies arises from the differences between Rayleigh (X-band) and non-Rayleigh scattering.

4. COMPARISONS TO MEASUREMENTS

Comparisons of the simulated radar bright-band profiles to the measured ones offer a direct check of the models as to their validity and accuracy. Illustrated in Figs.6 and 7 are the measurements of the radar reflectivity factors and mean Doppler velocities by EDOP and CRS on 7 July 2002 from 20:15:00 UTC to 20:25:00 UTC during CRYSTAL-FACE. The EDOP and CRS are the nadir-looking airborne Doppler radars operating at X and W bands respectively, mounted on NASA ER-2 aircraft during the CRYSTAL-FACE field campaign. A detailed description of the EDOP and CRS can be found in the literatures (Heymsfield et al. 1996; Li et al. 2004). The vertical profiles are also plotted in Figs.6 and 7 at selected locations along the flight line to provide examples of the vertical profiles. With a range resolution of 37.5 m, the signatures of the bright band are clearly detected by both radars at an altitude of around 4 km throughout the flight line. To make the measured profiles stable and less noisy, a smoothing procedure is used. This is done by first finding all the pairs of the X- and W-band profiles based on the criteria that the peaks of X-band are in the range of $Z_{\text{peak}}$ to $Z_{\text{peak}}+1$ (dB), and then averaging the selected profiles separately for X and W bands. It is worth noting that with such procedure, the stability of the measured radar mean profiles is dramatically improved. Shown in Fig.8 are the 4 EDOP (blue heavy-dotted lines) and CRS (red heavy-dotted lines) mean profiles that corresponding to the $Z_{\text{peak}}$ of 30, 32, 34 and 37 dB from top-left panel to bottom-right panel, respectively. Using the stratified-sphere melting particle model described earlier and assuming the Marshall-Palmer size distribution in rain, the simulated radar profiles (solid lines) are plotted and compared with the measured ones, as shown in Fig.8. The snow density used in our simulations is chosen as 0.1 g/cm$^3$, which is consistent with the findings of the study for the retrieval of the
Fig. 6 Measured radar reflectivity factors (top and middle panels) from EDOP and CRS nadir-looking airborne radar over a 130-km flight line over stratiform rain. The selected radar reflectivity profiles in the locations given by the dashed lines are shown in the bottom panel where the red and blue curves represent the EDOP and CRS radar reflectivity profiles, respectively.

Fig. 7 Measured mean Doppler velocities (top and middle panels) from EDOP and CRS for the same storm shown in Fig. 6. The selected mean Doppler velocity profiles are shown in the bottom panel where the red and blue curves represent the EDOP and CRS, respectively.

Fig. 8 Comparisons of simulated (solid) and measured (dotted) bright-band profiles at X (blue) and W (red) bands. The dashed lines are the simulated results without taking into account of attenuation, and the diamond-shaped data are the constructed un-attenuated W-band profiles based on the Doppler measurements.

Fig. 9 Plots of DFR vs. $D_0$ (left) and DDV vs. $D_0$ (right) for X- and W-band radars for rain as the shape factor ($\mu$) of the gamma particle distribution varies from 0 to 6.

The snow size distribution by use of dual-wavelength techniques for the same data (Liao et al., 2008). Because there is no particle breakup or aggregation assumed in the melting layer model, and also because the mass flux is constant within the melting layer, the particle size distribution (PSD) specified in rain can be uniquely converted to PSDs in the snow and melting layer regions. With the models being initialized in the way described earlier, the rain rate, which the Marshall-Palmer size distribution solely relies on, is the only free parameter in the simulation. In the comparisons depicted in Fig. 8, the rain rates
that give the best agreement between the simulated and measured profiles are 1.44, 1.71, 1.83 and 2.38 mm/h, respectively. As can be seen, the simulated radar bright bands are in excellent agreement with the measured ones at X band. They are not only matched well in the peaks of the bright band but also in the widths. However, the comparisons at W band are not as straightforward as those at X band in which the attenuation is negligibly small. This is because difficulties arise in the correction of attenuation caused by cloud water and water vapor. Although attenuation by hydrometeors (snow, melting snow and rain) is taken into account in our simulations, the attenuations from cloud water and water vapor are not included. Since neither cloud water nor water vapor is detectable, they are largely unknown. This introduces uncertainties in the higher-frequency radar retrieval. As illustrated in Fig.8, the simulated profiles (solid) at W band tend to agree with the measured ones (dotted) in shape but offsets in the magnitudes are clearly seen. To see if these offsets can be attributed to cloud water and water vapor attenuations at W band, we will conduct comparisons of non-attenuated radar profiles in rain between the model simulations and the reconstructed W-band profiles by use of the Doppler measurements.

By taking an advantage of simultaneous measurements of the Doppler velocities at X and W bands, we can derive the un-attenuated or true W-band radar profiles in rain. The differential Doppler velocity (DDV), which is defined as the difference of the Doppler velocities between X and W bands, depends only on the particle median volume diameter ($D_0$). This is also true of the radar dual-frequency ratio (DFR) in dB, which is equal to the difference of the radar reflectivity at X and W bands. Figure 9 depicts the relationships between DFR-$D_0$ (left) and DDV-$D_0$ (right) when the rain droplet size distribution is given by the gamma distribution. The $\mu$ in the plots is the shape factor of the gamma distribution, which is zero for the Marshall-Palmer size distribution (1948). Since the DDV is independent of the radar attenuation and also unaffected by air motion, $D_0$ can be estimated from the measured DDV (Tian et al., 2007; Liao et al., 2008). This in turn leads to a value of DFR from the differential Doppler-estimated $D_0$. The true radar reflectivity at W band is, by definition, the difference between the X-band reflectivity and the DFR, based on the assumption that attenuation at X band is negligible. This should be true for stratiform rain, particularly for the cases shown in Fig.8 where only light rain is present. The diamond-shaped data points in Fig.8 represent the non-attenuated radar profiles of rain at W band, derived from the DDV. The dashed curves refer to the non-attenuated W-band radar profiles generated from the models. There is a fairly good agreement between the non-attenuated radar rain profiles generated from the model on one hand and the estimated results on the other, implying good accuracy in simulating the W-band bright-band profiles. The differences of the W-band radar reflectivity profiles between the simulations (accounting for attenuation) and measurements can be explained by the W-band attenuation that results from cloud water and water vapor.

5. SUMMARY

To describe the snowflake-melting process in the transition of particles from snow to water, a stratified-sphere model is used to characterize the melting particles and compute the radar profiles at X and W bands in the melting layer. The effective dielectric constants, used at each layer of the stratified sphere, are derived from the realizations of the uniform snow-water mixtures by using CGFFT numerical method. As the fractional water content within the melting snow is expressed as an exponential equation, the simulations of the radar bright-band profiles are made at X and W bands under assumption that the rain follows the Marshall-Palmer size distribution. The simulated radar profiles are then compared to the X- and W-band Doppler radar measurements. While excellent agreement is found at X band, there are persistent offsets between the model and measured results at W-band. However, these
offsets can be explained by the attenuation caused by cloud water and water vapor at W band. This is confirmed by the comparisons of the radar profiles made in the rain regions where the un-attenuated W-band reflectivity profiles can be estimated through the X- and W-band Doppler velocity measurements. In particular, good agreement is shown for the un-attenuated profiles derived from the model-simulated results and the Doppler-derived results. Despite the difficulty in describing microphysical properties of hydrometeors in the melting layer, our simulations of the radar bright band made at X and W bands appear to be fairly accurate and suggest the usefulness of the stratified-sphere scattering model as well as the effective dielectric constants derived from mixed-phase particle realizations. The bright-band model described in this paper has potential to be used effectively for both radar and radiometer algorithms relevant to the TRMM and GPM satellite missions.

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