1. INTRODUCTION

As the raindrop shape is a key parameter in, e.g., the remote measurement of rain fall rates and nowcasting of precipitation using dual-polarization radars, the accurate knowledge of the oscillation behaviour of the raindrops is of great importance. In particular, it needs to be clarified whether the dynamic average shape (affected by oscillation) is the same as the one in static equilibrium while the latter is assumed in calculations of the rain fall rate from radar data.

Here we present the results of our experiments on raindrop oscillations of freely suspended drops with equivalent diameters of 1 to 7 mm floating inside the Mainz vertical wind tunnel at their terminal velocities. For this purpose a high speed digital video camera was used, which allows the continuous recording of the oscillation of individual raindrops for relatively long time intervals. The comparison of the measured equilibrium raindrop shape with the theoretical model of Beard and Chuang (1987) is presented. We show how the oscillation frequency and the time averaged axis ratio of raindrops depend on the drop size. This latter one is especially of high importance, as it is still an open question whether the dynamic and the static equilibrium axis ratios of raindrops with diameters ranging from 1 to 3 mm differ. Although a few publications dealt with this question (e.g. Andsager et al. 1999), there exists no reliable experimental verification until now on the consideration of Goddard and Cherry (1984) who predicted an enhancement in the axis ratio within this size range.

2. EXPERIMENTAL SETUP

The measurements were carried out in the vertical wind tunnel at the University of Mainz where water drops and other hydrometeors can be freely floated at their terminal velocities in a vertical air stream. Wind speeds up to 40 m/s are possible so that drops of all sizes between 40 µm and 8 mm can be investigated. In the measurements presented here freely suspended raindrops within the size range of 1 to 7.5 mm were floated at their terminal velocities in the wind tunnel. The drop oscillation was continuously recorded using a high-speed digital video camera (Motion-ProX, Redlake Inc.) with a recording speed of up to 2000 frames per second. The spatial resolution of the optical system was 24 µm which together with the millisecond time resolution allows the investigation of the raindrop oscillation with very high accuracy.

3. RESULTS AND DISCUSSION

3.1 Equilibrium drop shape

First the equilibrium shapes of drops of different sizes were measured and compared to those derived from the force balance model of Beard and Chuang (1987). We considered a drop to be in the “dynamic equilibrium” state if its actual axis ratio was equal to the average axis ratio. This average was determined from the temporal variation of the axis ratio as its mathematical mean. Figure 1 shows the measured dynamic equilibrium drop shapes together with the calculated shapes for drops of 0.6, 2.6, 5 and 7 mm equivalent diameters. It is apparent from the figure that the force balance model of Beard and Chuang (1987) correctly describes the equilibrium drop shape variation with size.
3.2 Oscillation frequencies and modes

The oscillation frequencies of the drops were determined from the temporal variation of the axis ratio. To achieve high accuracy, the frequency of the fundamental oscillation mode ($n=2, m=0$) was derived from the total time of several (typically 10) oscillation periods. In the cases, where a drop floated in the field of view of the camera for a sufficiently long time (typically for 1-2 seconds), it was possible to observe the beating of two oscillation modes in the temporal variation of the axis ratio, allowing to determine the frequencies of the higher mode oscillations.

Generally the formula from Rayleigh (1879) is used to calculate the fundamental oscillation frequency of a water drop falling at its terminal velocity. However, this formula is derived by postulating that the drops have a spherical form. As raindrops larger than 1 mm in diameter are no longer spheres (see Figure 1), the formula of Rayleigh cannot be valid for raindrops in the real atmosphere. Indeed, the deformed drop shape leads to a frequency shift as shown by the asymptotic analysis of Feng and Beard (1991). The oscillation frequencies corresponding to the ($n=2; m=0, 1$ and 2) modes for different drop sizes derived following the calculations of Feng and Beard (1991) are plotted on Figure 2 with thick, thin, and dashed lines, respectively. The frequencies of the fundamental mode determined from our frequency analysis on the floating drops are plotted by blue rectangles on Figure 2. It is apparent from Figure 2 that our measurements validate the theoretical considerations of Feng and Beard (1991). Thus, it is reasonable to determine hereinafter the drop size from the frequency of the fundamental mode, as it is more precise and reliable (<1% error) as that from the equivalent drop volume (~2.4% error), especially in the case when the drop floated in the tunnel for several minutes. In these cases the size of the drops is reduced by evaporation. For easier computation the Rayleigh frequency formula was used as the difference between the Feng and Beard (1991) and the Rayleigh frequency for the fundamental mode is negligible.

The empirical formula from the experiments of Nelson and Gokhale (1972) is also frequently used to calculate the oscillation frequency; see the green curve in Figure 2. This curve shows slightly higher oscillation frequencies compared to those from the models and from our measurements, however, this overestimation is not significant. The reason may lie in a systematic error in the determination of the drop size by Nelson and Gokhale (1972) which is not well-described in their article.

Nine raindrops with diameters between 4.2 and 7 mm that floated stable in the field of view of the camera for a sufficiently long time to investigate higher mode oscillations.
Because each oscillation mode has a different characteristic frequency corresponding to the given drop size (see Feng and Beard, 1991), it is possible to determine which oscillation modes are active by analyzing the beating frequency. The higher mode frequencies determined in this way for the nine raindrops are plotted with red symbols in Figure 2.

We found in our experiments that during the raindrop oscillation the (2,0) mode always exists (see the open symbols in Figure 2). The measured data points for drops with sizes between 4.5 and 7 mm corresponding to the higher mode frequencies calculated from the beating in the axis ratio variation lie on the curve corresponding to the (2,1) mode, indicating that the measured raindrops oscillated in this mode beside the fundamental mode. We experienced, however, the existence of the (2,2) mode and the non-existence of the (2,1) mode for drops with a diameter of 4.24 mm. Unfortunately, the difference in the frequency of the different modes decreases for small drop sizes, therefore, we could not observe the existence of higher modes for small drops. However, we can conclude from our experiments that the drops with various sizes oscillate in different modes which may depend on the drop size.

3.3 Axis ratio of the raindrops

The mean axis ratio was determined for each measured raindrop as the time-average of the varying axis ratio. The mean axis ratios as a function of the equivalent drop diameter calculated from the (2,0) mode oscillation frequency of the drop using the Rayleigh formula are plotted in Figure 3. The theoretical curve for the axis ratio calculated using the formula of Chuang and Beard (1990) is also shown in Figure 3 with a solid line, while the upper and lower limit of the model is indicated by dashed lines.

As can be seen from Figure 3, the measured time-average axis ratios fit well to the theoretical curve which verifies the method of the equivalent diameter determination from the oscillation frequency. Furthermore, this agreement verifies the force balance model of Beard and Chuang (1987) which derives the shape distortion from the balance of the internal hydrostatic and the external aerodynamic pressure and the surface tension force. It can be concluded from Figure 3, that the static and dynamic equilibrium drop shape is equal within the observed size range. Our findings contradict the predictions of Goddard and Cherry (1984). They estimated the raindrop size from the radar echo signal and corresponding disdrometer-obtained drop size distributions, and obtained higher axis ratio values in the size range between 1 and 3 mm (blue curve in Figure 3) than those from the force balance model of Beard and Chuang (1987).

During oscillation the drop has different shapes, i.e. different axis ratios, which has to be considered in the backscatter ratios for the radar signal evaluations. If the drop floated in the field of view of the camera for such a long time to record several periods of beating, a reliable histogram of the axis ratio variation could be made. Figure 4 shows the distribution of the axis ratio variation for a 4.24 mm diameter drop. The distribution of the axis ratio is uniform, which is the consequence of the coexistence of different oscillation modes.
The basic problem with the experiments in the past regarding the drop oscillation was that the time evolution of the drop shape under oscillation was not recorded as a continuous time series. From our experiments, however, it was possible to determine the typical oscillation amplitude of the axis ratio, which is very important in the uncertainty analysis of the precipitation nowcasting from radar signals. To the best of our knowledge, there exists no theoretical prediction on the amplitude variation of the axis ratio of drops, therefore, we can present only our speculations. The oscillation is induced by the vortexes shedding from the rear of the drop, and the energy of the vortex shedding increases with the drop size.

The oscillation amplitude is proportional to the energy which induces the oscillation, thus, the larger the drop, the larger is the amplitude of the oscillation. Our measurements verify this consideration, as can be seen on Figure 2, where the amplitude of the axis ratio is plotted as the function of the equivalent drop diameter. Moreover, we found that the best fit to the amplitude vs. drop diameter curve is a second order polynomial fit.

4. BIBLIOGRAPHY


ACKNOWLEDGEMENT

These studies were financed by the Deutsche Forschungsgemeinschaft (DFG) under project BO 1829/1-1/2 within the AQUARadar program and also supported by the Max-Planck Institute for Chemistry.