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1. INTRODUCTION

The EMPM (Explicit Mixing Parcel Model) predicts the evolving in-cloud variability of temperature and water vapor mixing ratio due to entrainment and finite-rate turbulent mixing using a 1D representation of a rising cloudy parcel (Krueger et al. 1997). The 1D formulation allows the model to resolve fine-scale variability down to the smallest turbulent scales (about 1 mm). The EMPM calculates the growth of thousands of individual cloud droplets based on each droplet's local environment (Su et al. 1998).

How do entrainment and mixing affect droplet spectral evolution in the EMPM? The following sequence of the events is illustrated in Fig. 1 for an isobaric entrainment and mixing event. The parcel first ascends adiabatically above cloud base, while the droplets grow by condensation. When entrainment occurs, the subsaturated entrained air replaces a a same-sized segment of the cloudy parcel. The cloudy air and the newly entrained air undergo a finite rate turbulent mixing process. During this process, many droplets encounter the entrained subsaturated air, resulting in partial or even total evaporation of some droplets.

We used the EMPM to investigate the impact on droplet spectra evolution in cumulus clouds of the following aspects of entrainment and mixing:

- **Parcel trajectory after entrainment:** Isobaric versus ascending.
- **Entrained CCN concentration:** Zero, half cloud base concentration, or full cloud base concentration.

We were motivated by aircraft measurements in cumulus clouds of cloud droplet number concentration (N) and mean volume radius (r_v) , aver-



Figure 1: A parcel undergoing isobaric mixing is represented by a 1D domain in the EMPM. The parcel's internal structure evolves due to discrete entrainment events and turbulent mixing (turbulent deformation and molecular diffusion). Droplets evaporate based on each droplet's local environment.

aged over 10-m intervals, normalized by their adiabatic values, and plotted on a diagram with coordinates N/N_a and $V/V_a = r_v^3/r_{va}^3$. The product of the coordinates is the LWC normalized by its adiabatic value. Such a diagram (from Burnet and Brenguier 2007) for cloud traverses about 1500 m above cloud base for a case during SCMS (Small Cumulus Microphysics Study) is shown in Fig. 2. The challenge is to explain the observed distributions.

Burnet and Brenguier proposed that isobaric

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Figure 2: *N*-*V* diagram from SCMS for 10 August 1995. Plotted are 909 cloud samples, each of 10-m length. From Burnet and Brenguier (2007).

mixing, combined with buoyancy sorting, can explain the the observed distributions of N and r_v in cumulus clouds. However, we propose that additional processes (ascent of entrained air and entrainment of CCN) are likely to be important.

To explore the range of potential N-V distributions that might be encountered in cumulus clouds and to relate them to cloud processes, we applied the EMPM to a variety of realistic entrainment and mixing scenarios. The consequences of parcel trajectory after entrainment (isobaric versus ascending), and entrained CCN concentration on N-V distributions in entraining, non-precipitating cumulus clouds as predicted by the EMPM are presented in Section 2. Conclusions follow in Section 3.

2. ENTRAINMENT AND MIXING IN THE EMPM

2.1 Isobaric mixing

Figure 3 shows the sequence of states involved in isobaric mixing in the EMPM after an entrainment event. The states are numbered from 1 to 4. State 1 is the result of adiabatic ascent from cloud base. The variability of the droplet number concentration at this time is due to the small number of droplets (about 100) in each 1-m segment. State 2 is due to the initial breakdown of the entrained blob into smaller segments, with little droplet evaporation. This reduces N and LWC by dilution, but does not decrease r_v .

Between states 2 and 3, droplets evaporate until local saturation is achieved. This reduces the local r_v , but does not change the local N unless some droplets totally evaporate. In this case, almost no droplets totally evaporate. The blue line is the so-called "homogeneous" mixing line. It indicates all possible values of (N,V) in *saturated* mixtures of entrained and adiabatic (undiluted cloud-base) air in which no droplets have totally evaporated. ¹ Therefore, the N-V distribution moves downwards towards the blue line between states 2 and 3. In state 3, all mixtures are again saturated, due to droplet evaporation. The rate at which the adjustment to saturation occurs is limited by the rate of turbulent mixing in this case.

Between states 3 and 4, the resulting saturated parcels mix. Because the blue line is also a mixing line for saturated parcels, the N-V distribution converges towards its domain average. In state 4, the parcel is once again statistically uniform.

Burnet and Brenguier used a simple mixing model to demonstrate that isobaric entrainment and mixing events can produce (N, V) pairs anywhere on the diagram between the "homogeneous" mixing line and N = 0. This result agrees with the EMPM results shown in Figure 3.

Figure 4 conceptually illustrates the sequence of states involved in isobaric mixing after an entrainment event for two parcels based on the analysis of EMPM results such as those shown in Fig. 3. The entrained air fraction is greater for the "blue" parcel (0.5) than for the "red" parcel (0.3). As a result, the LWC of the "blue" parcel is less than that of "red" parcel, both immediately after entrainment (state 2), and after saturation adjustment (state 3). In this case, no droplets totally evaporate, so the state 3 N - V coordinates for both parcels lie on the same mixing line. Mixing between these the two parcels produces state 4,

¹For entrainment into cumulus clouds, the mixing line depends primarily on the relative humidity (RH) of the entrained air.



Figure 3: N-V diagram for isobaric mixing in the EMPM after an entrainment event. Each point is a 1-m average. Plotted in each panel are points from 11 "traverses" of the 80-m EMPM domain during each 8.25-s interval ending at the indicated time.



Figure 4: Entrainment and isobaric mixing for two parcels. No droplets totally evaporate.



Figure 5: Entrainment and isobaric mixing when some droplets completely evaporate. The mixing parameter α is given for each mixing scenario.

which also lies on the mixing line.

Figure 5 is like Fig. 4 except that in this case some droplets completely evaporate between states 2 and 3, so that N decreases *after* entrainment. If V does not change during evaporation, the process is called "extreme inhomogenous mixing, and each droplet either completely evaporates, or does not evaporate at all. As before, the parcel is saturated with LWC = NV when it reaches state 3.

Morrison and Grabowski (2008) proposed the following general relationship betwen N_f , the final droplet concentration after turbulent mixing and evaporation, and N_i , the droplet concentration after entrainment (for a parcel model) or after transport (for an Eulerian grid volume):

$$N_f = N_i \left(\frac{\mathsf{LWC}_f}{\mathsf{LWC}_i}\right)^{\alpha},\tag{1}$$

where LWC_f and LWC_i are the final and initial liquid water contents, and $0 \le \alpha \le 1$. For so-called homogeneous mixing, $\alpha = 0$, and for extreme inhomogeneous mixing, $\alpha = 1$.

Solving (1) for α gives

$$\alpha = \frac{\log(N_f/N_i)}{\log(\mathsf{LWC}_f/\mathsf{LWC}_i)}.$$
 (2)

Equation (2) applies equally well to ratios of normalized quantities. We used (2) to calculate α for



Figure 6: Time sequence of domain averages for isobaric mixing in the EMPM (80-m domain) after 7 sequential entrainment events. The mixing parameter α is given for each entrainment event.

each mixing scenario in Fig. 5, and for each entrainment and isobaric mixing event in the EMPM simulation shown in Fig. 6. The results suggest that, for given entrained air properties and mixing time scale, α increases as the LWC decreases.

Schlüter (2006) analyzed a set of more than 100 EMPM simulations of entrainment and isobaric mixing that covered a wide range of entrained air properties and mixing time scales. We have used her results to calculate α for each of the EMPM simulations. We anticipate that further analyis of her results will provide some guidance for parameterizing α in cloud-resolving models that do not resolve the entrainment and mixing process.

2.2 Ascent with and without entrained CCN

Figure 7 presents the distributions of the domain averages of two EMPM simulations of isobaric mixing in a 20-m domain with 7 sequential entrainment events. One had no entrained CCN, while the other entrained CCN. Note that entrained CCN have no impact when the mixing is isobaric.

The two plots in Fig. 8 show the dramatic impact of entrained CCN in an *ascending* parcel (80-m domain) with sequential entrainment events. Without entrained CCN (left panel), r_v^3 grows to



Figure 7: Time sequence of domain averages for isobaric mixing in the EMPM (20-m domain) after 7 sequential entrainment events. Left: No entrained CCN. Right: with entrained CCN.



Figure 8: Like Fig. 7 except for ascent in an 80-m EMPM domain.



Figure 9: Left: Entrainment, mixing, and condensation (C) for a parcel that ascends after each entrainment event but entrains no CCN. Right: Entrainment, mixing, activation (A), and condensation (C) for a parcel that ascends after an entrainment event that entrains CCN.

150 percent of adiabatic at the highest level (1500 m above cloud base), while N decreases to 25 percent of adiabatic. When CCN are entrained at cloud base concentrations (right panel), r_v^3 decreases to about 40 percent of adiabatic, while N only slightly decreases, to about 90 percent of adiabatic.

If a cloudy parcel ascends during entrainment and mixing, the relative humidity of the entrained air will increase, thereby shifting the mixing line upwards and increasing the LWC ("feeding"). If no CCN are entrained and no droplets totally evaporate, N will remain constant after entrainment, so that r_v will also increase. This "weed and feed" scenario is illustrated in the left panel of Fig. 9.

Due to ascent and adiabatic cooling, newly entrained air may become supersaturated and some of the entrained CCN may be activated, thereby increasing N ("seeding") but decreasing r_v (for constant LWC). This "weed, seed, and feed" scenario is illustrated in the right panel of Fig. 9.

Figure 10 shows the time series of all 10-m averages for an EMPM simulation in a 200-m domain without entrained CCN. Compared to the domain-averaged results, the 10-m averages are much more variable (and realistic) because the entrained air fraction in each 10-m segment is determined by the EMPM's stochastic mixing process, rather than being specified. As a result, the 10-m averages from the 200-m domain results can be directly compared to aircraft measurements, such as those shown in Fig. 2.

Figure 11 shows the time series of all 10-m averages for an EMPM simulation in an 80-m domain with entrained CCN at one half of cloud base concentrations, while Fig. 12 shows the same for an EMPM simulation with entrained CCN at cloud base concentrations.

3. CONCLUSIONS

Entrainment followed by *isobaric* mixing reduces the droplet number concentration by dilution ("weeding") and the LWC and mean volume radius by droplet evaporation. As long as no droplets completely evaporate, the entrained air fraction determines N, and mixtures of entrained and adiabatic (undiluted cloud-base) air define the so-called "homogeneous" mixing line on the N- r_v^3 diagram.

If a cloudy parcel ascends during entrainment and mixing, the RH of the entrained air will increase, thereby shifting the mixing line upwards and increasing the LWC ("feeding"). If N remains constant, r_v will also increase. Due to ascent and adiabatic cooling, newly entrained air may become supersaturated and some of the entrained CCN may be activated, thereby increasing N ("seeding") but decreasing r_v (for constant LWC).

These (and other) comparisons between EMPM results and the observations of Burnet and Brenguier (2007) suggest that distributions of N and V similar to those observed can be produced in an ascending parcel by entraining air with intermediate CCN concentrations.

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Figure 10: 10-m averages for an EMPM simulation in a 200-m ascending domain without entrained CCN. Left: All values. Right: Values for a short time interval, similar to what would be sampled by an aircraft traverse.



Figure 11: Like Fig. 10 but for entrained CCN at one half of cloud base concentrations.



Figure 12: Like Fig. 10 but for entrained CCN at cloud base concentrations.