# RETRIEVING THREE-DIMENSIONAL CLOUD STRUCTURE USING A TOMOGRAPHY METHOD

Dong Huang<sup>1</sup>, Yangang Liu<sup>1</sup>, and Warren Wiscombe<sup>1,2</sup>

<sup>1</sup>Brookhaven National Laboratory, Atmospheric Sciences Division, Upton, NY 11973, U.S.A. <sup>2</sup>NASA Goddard Space Flight Center (code 913), Greenbelt, MD 20771, U.S.A.

## 1. INTRODUCTION

Three-dimensional distributions of cloud water are needed for studying cloud microphysics and atmospheric radiation, and for validating cloud-resolving and large-eddy-simulation models. In addition to the costly active remote sensing technique like radar, cloud tomography offers the promise of retrieving 3D cloud water distributions using multi-beam microwave emission measurements (Warner et al., 1985&1986). The method was proposed in the 1980s, but neither the technology nor the cloud models were mature enough to make any practical application. Now, the time is ripe for a renewed push. We have created a Tomography Simulator with simulated clouds and simulated microwave radiometers to show the feasibility of the cloud tomography method.

#### 2. MATHEMATIC FORMULATION

The radiative transfer equation relating the microwave radiation intensity to the atmosphere state is:

$$I(\Omega_i) = I_{\infty} \tau(\Omega_i, 0, \infty) + \int_0^\infty B(T) \alpha(s, \Omega_i) \tau(\Omega_i, 0, s) ds,$$
(1)

where  $I(\Omega_i)$  is the intensity of radiation reaching a radiometer from direction  $\Omega_i$ ;  $I_{\infty}$  is the intensity of the cosmic background radiation; B(T) is the Planck function at temperature T;  $\alpha$  is the absorption

coefficient of cloud liquid water determined by the atmosphere state; and

$$\tau(\Omega_i, s_1, s_2) = \exp[-\int_{s_i}^{s_2} \alpha(s, \Omega_i) ds]$$
 is the

transmission between two points  $s_1$  and  $s_2$  along direction  $\Omega_i$ .

Given a total number of m rays, Eq. (1) can be discretized by dividing a field, which is large enough to contain the cloud, into  $n=N^3$  ( $N^2$  for a 2D slice) equal size volume pixels to yield the following matrix equation (Huang et al., 2007):

$$\mathbf{A}\mathbf{x} = \mathbf{b} \,, \tag{2}$$

where  $\mathbf{x}^T = (\alpha_1, \alpha_2, \cdots, \alpha_n)$  is the vector of absorption coefficients of cloud liquid water;  $\mathbf{b}^T = (b_1, b_2, \cdots, b_m)$ , is the vector of measurements,  $b_i$  equals the right side of Eq.(3); and  $\mathbf{A} = (a_{ij})$  is the mxn kernel matrix that representing the radiative transfer operator. When cloud is found in the retrieval to occupy only part of the field or the information of cloud boundary is available from other measurements like Radar, the retrieval process can be refined with a smaller field to get a better spatial resolution.

## 3. RETRIEVAL ALGORITHM

For a limited-angle tomographic problem like that of the cloud tomography technique, an ideal, unambiguous retrieval would require the data and the kernel

matrix **A** to be free of noise and each cloud element to be scanned from all directions (Olson, 1995). Because both conditions are impossible to meet in reality, multiple solutions may satisfy the same radiometric measurements, and special regularization techniques beyond the standard method of least squares are needed to deal with this problem.

Following the Bayesian theorem, we propose an algorithm that can use either the smoothness constraint or the nonnegativity constraint, or a double-side constraint defined by an initial estimate of the retrieval, or a combination of any of the above. Essentially, the algorithm solves the following minimization problem (Huang et al., 2008):

$$\min_{\mathbf{x}} \left\{ \left\| \mathbf{A}' \mathbf{x} - \mathbf{b}' \right\|_{2}^{2} \right\}, \text{ subject to } \mathbf{x} \ge 0,$$
 (3)

where  $\mathbf{A}' \equiv \mathbf{A}^T \mathbf{A} + \lambda \mathbf{L}^T \mathbf{L} + \tau \mathbf{Q}^{-2}$ ,  $\mathbf{b}' \equiv \mathbf{A}^T \mathbf{b} + \tau \mathbf{Q}^{-2} \mathbf{x}_b$ .  $\mathbf{L}$  is the matrix of the two-dimensional first derivative operator;  $\mathbf{Q}$  is the error co-variance matrix of the initial estimate  $\mathbf{x}_b$ ;  $\lambda$  and  $\tau$  are the regularization parameters determining the amount of the smoothness and double-side constraints imposed on the retrievals. The initial estimate  $\mathbf{x}_b$  can be specified by using either a scaled adiabatic profile of cloud liquid water content or another independent observation such as the cloud liquid water field derived from a dual-frequency radar.

## 4. SIMULATIONS

A two-dimensional 5 Km wide and 1.5 Km high slice of cloudy atmosphere is taken from the simulations of a Large Eddy Simulation (LES) model driven by the data

from the Atlantic Stratus Experiment. The original high-resolution LES simulation is degraded to an image of 20 by 20 pixels (250-meter horizontal and 75-meter vertical resolution). Four simulated radiometers of 0.3 K noise level and 2-degree beam width are placed equally on the ground along a line of 10 Km (Figure 1). Each radiometer scans the upper plane within 85° elevation of zenith at a 0.4° increment. This scanning strategy results in a total number of 800 rays hitting the 5 Km by 1.5 Km area.

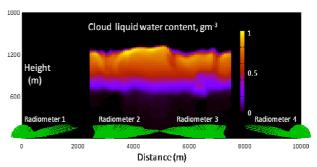
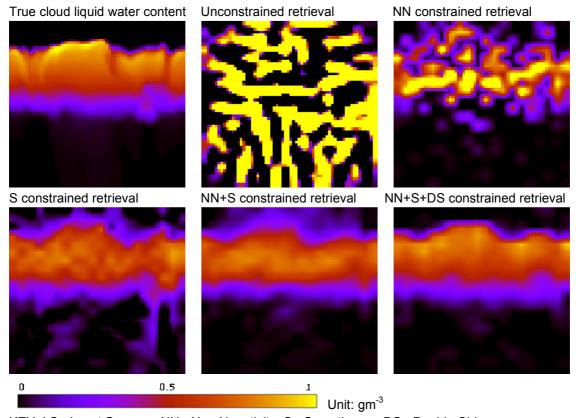


Figure 1. An example of a four-radiometer cloud tomography setup. Each radiometer scans the upper plane to within 5° of the ground; the scans are every 0.4° in angle. The lengths of the green lines from each radiometer are proportional to the simulated brightness temperatures in that direction. The atmospheric background is assumed to be 20 K.

The simulated tomographic data are then inverted using the algorithm described in Section 3. We first examine the effects of adding different constraints on the retrieval of the four-radiometer setup shown in Figure 1 for the stratocumulus cloud. As shown in Figure 2, the retrieved cloud from the standard least squares method shows very unrealistic spatial patterns of the cloud liquid water content. The addition of the non-negativity and smoothness constraints helps to capture the location and spatial extent of the cloud, but gives poor retrievals at cloud edges. The incorporation of a double-side constraint (based on



KEY: LS - Least Squares; NN - Non-Negativity; S - Smoothness; DS - Double-Side

Figure 3. The retrieved cloud liquid water content from the cloud tomography simulation shown in Figure 1 using various types of constraints. The true field is also shown as a reference.

scaled adiabatic profiles) produces the best cloud tomography retrieval. It not only accurately captures the location and extent of the stratocumulus cloud, but also accurately reproduces the cloud edges.

We then perform a group of sensitivity studies to identify the key factors that determine the retrieval accuracy of cloud tomography. When more radiometers and/or more scanning angles are used, and/or the radiometer beam width is reduced, and/or when a coarser output resolution is acceptable, a better retrieval can be obtained. The uncertainty in the ancillary data such as environment temperature and water vapor mixing ratio also impacts the retrieval, but the impact is

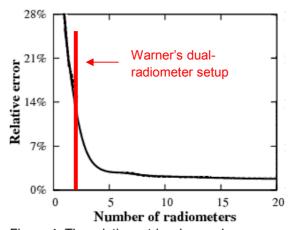


Figure 4. The relative retrieval error decreases when more radiometers are used. Warner's dual-radiometer setup is indicated by a vertical red line. Apparently it is not the optimal choice for this case.

considerably small over the range of uncertainty levels provided by radiosonde or sounding measurements. Among the factors the number of ground radiometers used appears to be the most critical one, as shown in Figure 4. There exists a critical point, say 4, beyond which adding more radiometers doesn't improve the retrieval much. This suggests that in this situation other types of information may be needed to further improve the retrieval, for example range-resolved information from a radar.

Furthermore, we show that the addition of radar data can improve the retrieval even further (Figure 5). The radar data are simulated using a Mie scattering code at the 35G and 94G frequencies and are imposed with a 0.5 dBz Gaussian noise. The difference between the differential attenuation at the two frequencies is converted to cloud liquid water content using the method of Hogan et al. (2005). The derived cloud field is then used as an initial estimate to constrain the retrieval using Eq. (3). The simulations show that the combination of data from two radiometers and one dual-frequency radar obtains the same accuracy as using eight radiometers.

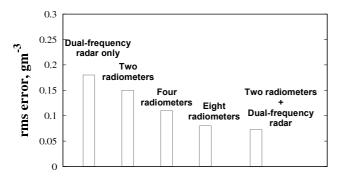


Figure 5. The rms errors for the retrieved cloud liquid water content using different combinations of radiometers and radar.

#### **ACKNOWLEDGEMENTS**

This research is supported by the DOE Atmosphere Radiation Measurement program under Contract DE-AC02-98CH10886.

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